

Shock layer thickness and optimum linear velocity in displacement chromatography

Jie Zhu and Georges Guiochon*

Department of Chemistry, University of Tennessee, Knoxville, TN 37996-1501 (USA) and Division of Analytical Chemistry, Oak Ridge National Laboratory, Oak Ridge, TN 37831-6120 (USA)

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ABSTRACT

The width of the mixed zones between two successive bands in the isotachic train represents the loss in recovery yield achieved in displacement chromatography. Intuitively, this width depends on the mobile phase flow velocity, but no systematic study of this effect has yet been performed. On the other hand, constant pattern behavior and the theory of shock layer are well known in chemical engineering. Using this approach, and assuming competitive Langmuir isotherm behavior, an analytical equation is derived which relates the shock layer thickness (SLT) in displacement chromatography and the column design and operating parameters. Using this equation, it is possible to investigate the dependence of the SLT between two consecutive bands in the isotachic train on the mobile phase velocity, the concentration and the retention factor of the displacer and the separation factor of the two components. In displacement chromatography, the optimum mobile phase linear velocity (u_{opt}^S) for minimum shock layer thickness, or maximum recovery yield depends not only on the coefficients of axial dispersion and mass transfer resistance of the two components, as does the optimum mobile phase velocity (u_{opt}^L) in linear chromatography, but also on the retention factor and the concentration of the displacer. The results of the study of this analytical equation are in excellent agreement with those of numerical calculations.

INTRODUCTION

Although suggested by Tswett [1], displacement chromatography was really introduced by Tiselius and Claesson [2] and Spedding [3] in the 1940s. Glückauf [4] developed its theory soon afterwards in the case of Langmuir isotherm behavior. He also recognized that a self-sharpening effect of thermodynamic origin (due to the non-linear behavior of the isotherm) counteracts the band spreading due to the finite rate of mass transfer [5]. Later, Helfferich and Klein [6] and Rhee and Amundson [7] completed the theoretical study of displacement in the frame-

work of the ideal model. No general theory is available in the equilibrium-dispersive and kinetic models, although numerical solutions have been published and studied [8–10]. Although displacement has been widely used in the past for preparative applications [2,3,11], it fell into oblivion as a practical technique shortly after World War II. In spite of systematic efforts [12,13] to reintroduce it as a preparative method of separation, its renaissance is considerably slowed by the lack of understanding of many issues of critical importance in actual practice. One of them is related to the mixed zone between consecutive bands in the isotachic train.

The ideal model predicts that two consecutive bands are separated by a concentration shock. There are no mixed zones between consecutive bands, hence complete recovery of the feed as separated products is possible. This ideal situa-

* Corresponding author. Address for correspondence: Department of Chemistry, University of Tennessee, Knoxville, TN 37994-1501, USA.

tion is, of course, not observed with actual columns. A mixed zone of finite width takes place between consecutive bands. Calculations show that, under experimental conditions giving a high value of the production rate, the recovery yield achieved in displacement chromatography because of the finite width of this mixed zone is as low as, or even lower than, the recovery yield in overloaded elution chromatography [14]. Admittedly, the isotachic train is not yet formed under these conditions, which contributes much to the reduced yield. Nevertheless, as most practitioners seem to prefer performing displacement separations under isotachic conditions and would like to improve recovery yields as much as possible, it is important to study the parameters which control the width of these mixed bands. Unfortunately, little information and much contradiction is found on this topic in the literature.

Horváth and co-workers [12,13,15,16] reported that the resolution between adjacent bands decreases with increasing flow-rate. They suggested that displacement development should be carried out at a flow-rate 2–10 times lower than that used in elution under linear conditions [13]. They also reported that operating the column at high flow-rates, in order to increase the production rate, is done at the expense of a markedly decreased recovery yield [13], and recommended flow-rates much lower than those used typically in elution with similar columns. This result is in agreement with the observation made by Cardinali *et al.* [17] that the overlap between the last component and the displacer decreases markedly with decreasing flow-rate. On the other hand, Subramanian and co-workers [8,18] reported an opposite result. They observed that the flow-rate does not affect much the band profile in the flow-rate range 0.1–1 ml/min (for a 4.6 mm I.D. column). Similarly, from the results of Cardinali *et al.* [17], the degree of overlap between successive bands of the isotachic train seems to be little affected by the change in flow-rate made by these workers. These apparent contradictions deserve some clarification.

We know that the concentration shocks predicted by the ideal model for the chromatograms obtained with infinitely efficient columns cannot actually take place. Real columns have a finite

efficiency. The concentration shocks that non-linear equilibrium isotherms tend to build up are eroded by the effects of axial dispersion and of the finite rate of the mass transfer kinetics. Very steep fronts, in which the concentrations of one or several components change rapidly, take place instead. These regions are called shock layers. When adsorption isotherms are convex upwards, as they should be in displacement chromatography, the breakthrough curves observed in frontal analysis have a very steep front [11,19]. Similarly, the boundaries between successive zones in displacement chromatography are also very steep. When the isotachic train has been formed, it propagates unchanged. This means that a constant pattern or steady-state, dynamic equilibrium is reached between the self-sharpening trend driven by thermodynamics and the dispersive effects of the finite column efficiency. The boundaries between two successive bands are binary shock layers. The simplest and most useful model for the profiles of these layers has been derived and studied by Rhee and co-workers [20–23]. This theory results into a fairly simple expression for the shock layer thickness (SLT) in the case of Langmuir adsorption behavior [23].

In a previous paper [24], we presented the results of a theoretical and experimental investigation of the dependence of the SLT in single component frontal analysis on the two main operating parameters which control it, the mobile phase velocity and the height of the concentration step injected into the column. The SLT is simply related to the column height equivalent to a theoretical plate (HETP), the height of the concentration step and the isotherm. While there is an optimum linear velocity, u_{opt}^L , for minimum HETP in linear chromatography, there is an optimum linear velocity, u_{opt}^S , for minimum SLT in frontal analysis. Both velocities are related, but may be very different. u_{opt}^S depends strongly on the limit retention factor or initial slope of the isotherm [24]. Therefore, a similar investigation made in displacement chromatography could help in understanding the factors controlling the thickness of the intermediate, mixed zones between successive bands in the isotachic train in displacement chromatography.

The theory of multi-component shock layers

has been developed by Rhee and Amundson [23] and applied by them to the study of shock layer profiles in multi-component frontal analysis. Recently, Guiochon and co-workers [24–27] used these results to study the accuracy of isotherm measurements by binary frontal analysis, the optimization of the experimental conditions and some related problems. In this paper, we apply the analytical equation derived by Rhee and Amundson for the calculation of SLT to the study of the width of intermediate mixed zones in displacement chromatography, for mixtures of components following the competitive Langmuir isotherm model. We discuss the influence on the SLT of the mobile phase velocity, the concentration and the retention factor of the displacer and the separation factor between the two components.

THEORY

We first recall the definition of shock layers, and briefly explain how the work of Rhee and Amundson [21] on single-component shock layers can be extended to binary shock layers [23]. Then, we derive an analytical equation relating the SLT in displacement chromatography and the experimental conditions. Throughout the discussion, we assume competitive Langmuir isotherms. For the sake of simplicity, the isotherm is written as

$$q_i = \frac{q_s \Gamma_i}{1 + \Gamma_1 + \Gamma_2} \tag{1}$$

where q_i is the concentration of component i at equilibrium in the stationary phase, q_s is the saturation capacity of the stationary phase and $\Gamma_i = b_i C_i$, b_i being the coefficient of the Langmuir isotherm and C_i the mobile phase concentration of i .

Because the shock layer theory, as all theoretical models of chromatography so far, consider the column as radially homogeneous, and hence neglects the possible influence of radial perturbations in the packing density, we consider only the abscissa in discussing column properties. Similarly, the mobile phase flow velocity, not its flow-rate, is significant in the study of the mass transfer resistance. Therefore, we give the shock layer thickness in time and distance units, not in

volume as suggested by a reviewer. Multiplication by the appropriate value of the column cross-sectional area would supply easily these data, when needed.

The SLT in single-component frontal analysis

In frontal analysis, a stream of constant concentration is injected into the column. Assuming that the column is initially empty, the front or breakthrough curve propagates along the column at a constant velocity, U_s [4–7]:

$$U_s = \frac{u}{1 + K} \tag{2}$$

with

$$K = \frac{k'_0}{1 + \Gamma_0} \tag{3}$$

where u is the mobile phase velocity, k'_0 is the retention factor, $a = k'_0/F$ and b are the parameters of the Langmuir isotherm, F is the phase ratio and C_0 is the step concentration injected. The SLT is the distance, $\Delta\eta_x$, between two concentrations C_1^* and C_2^* inside the column (Fig. 1), or the time, $\Delta\eta_t$, separating the elution of these two concentrations:

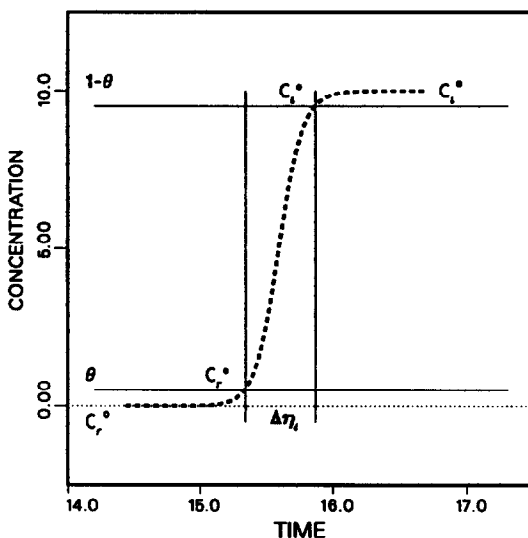


Fig. 1. Definition of the shock layer thickness (SLT). The curve shows the concentration profile, $C(t)$, of the breakthrough curve in single-component frontal analysis obtained for the injection of a step from $C = 0$ to $C = C_0 = 10$ mM. Concentration in mM, time in min.

$$C_r^* = \theta C_0 \quad (4a)$$

$$C_1^* = (1 - \theta) C_0 \quad (4b)$$

where θ is an arbitrary number lower than 0.5, usually between 0.01 and 0.05. The SLT can be calculated by introducing the dimensionless moving coordinate:

$$\xi = \frac{x - tU_s}{L} \quad (5)$$

The SLT is given by

$$\Delta\xi = \xi(C_r^*) - \xi(C_1^*) \quad (6a)$$

$$\Delta\eta_x = L \Delta\xi \quad (6b)$$

$$\Delta\eta_t = \frac{L}{U_s} \cdot \Delta\xi \quad (6c)$$

where L is the column length.

Rhee and Amundson [21] have shown that the SLT is given by the following equation:

$$\Delta\xi = \left[\frac{D_a(1+K)}{KuL} + \frac{u}{(1+K)k_f L} \right] \frac{\Gamma_0 + 2}{\Gamma_0} \cdot \ln \left| \frac{1-\theta}{\theta} \right| \quad (7a)$$

$$\Delta\eta_x = \left[\frac{D_a(1+K)}{Ku} + \frac{u}{(1+K)k_f} \right] \frac{\Gamma_0 + 2}{\Gamma_0} \cdot \ln \left| \frac{1-\theta}{\theta} \right| \quad (7b)$$

$$\Delta\eta_t = \left[\frac{D_a(1+K)^2}{Ku^2} + \frac{1}{k_f} \right] \frac{\Gamma_0 + 2}{\Gamma_0} \cdot \ln \left| \frac{1-\theta}{\theta} \right| \quad (7c)$$

where D_a is the axial dispersion coefficient, including the effects of molecular axial diffusion, tortuosity and eddy diffusion [28,29], and k_f is a lumped mass transfer coefficient [20-23,30]. $\Gamma_0 = bC_0$ is the dimensionless sample injection concentration.

The SLT in multi-component displacement chromatography

When the formation of the isotachic train is achieved in displacement chromatography, all the bands migrate at the same velocity as the displacer front. A constant pattern, *i.e.*, an asymptotic solution, has been reached and there

is a shock layer between two consecutive bands. The train moves at the velocity

$$U_{sd} = \frac{u}{1+K_d} = \frac{u}{1 + \frac{k'_{0,d}}{1+\Gamma_d}} \quad (8)$$

The plateau concentration of each band in the isotachic train is determined by the displacer concentration. It is given by

$$C_{p,i} = \frac{1}{b_i} \left[\frac{a_i}{a_d} (1 + \Gamma_d) - 1 \right] \quad (9)$$

where $\Gamma_d = b_d C_d$ and b_d and C_d are the Langmuir isotherm parameter and the injection concentration of the displacer, respectively.

All the previous results are given by the ideal model. They are still valid with actual columns having a finite efficiency. However, the rear boundary of each band in the isotachic train is mixed with the front boundary of the following band (Fig. 2). The SLT is given by the same definition as for a single component (compare Figs. 1 and 2). By replacing the plateau injection concentration, C_0 , in eqn. 6 by the plateau

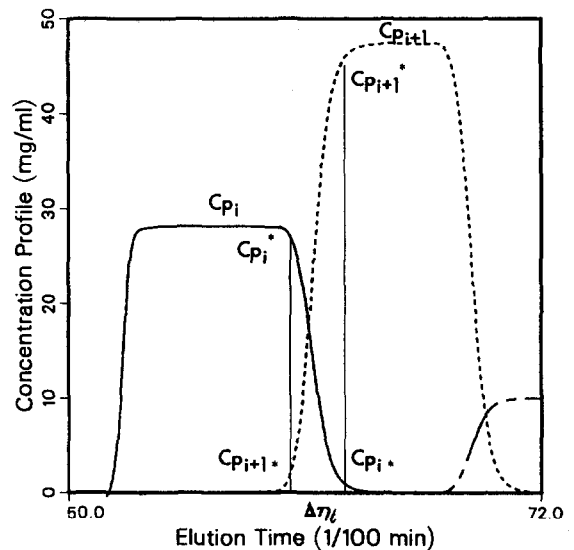


Fig. 2. Definition of the shock layer thickness (SLT) between two bands of the isotachic train in displacement chromatography. C_{p1} and C_{p2} are the plateau concentrations; $C_{p,i}^* = (1 - \theta)C_{p,i}$, $C_{p,i+1}^* = \theta C_{p,i}$, $C_{p,i+1}^* = (1 - \theta)C_{p,i+1}$, $C_{p,i+1}^* = \theta C_{p,i+1}$. $\Delta\eta_t$ is the shock layer thickness in time unit.

concentration, $C_{p,i}$, we obtain for the SLT in multi-component displacement chromatography

$$\Delta\xi = \xi(C_{p,i}^*) - \xi(C_{p,i,*}) = \xi(C_{p,i+1}^*) - \xi(C_{p,i+1,*}) \tag{10}$$

where

$$C_{p,i}^* = \theta C_{p,i}; C_{p,i,*} = (1 - \theta)C_{p,i} \tag{11a}$$

$$C_{p,i+1}^* = \theta C_{p,i+1}; C_{p,i+1,*} = (1 - \theta)C_{p,i+1} \tag{11b}$$

Rhee and Amundson [23] have shown that the concentration profiles in the shock layers in each mixed zone are described by the following partial differential equation:

$$-\frac{\lambda}{PeSt} \cdot \frac{d^2 C_i}{d\xi^2} + \left[\frac{1}{Pe} + \frac{\lambda(1-\lambda)}{St} \right] \frac{dC_i}{d\xi} = \lambda F\mathcal{F}(C_i, C_{p,i}, C_{i+1}, C_{p,i+1}) \tag{12}$$

where i is the rank of the component in the train, between 1 and n . The function $\mathcal{F}(C_i, C_{p,i}, C_{i+1}, C_{p,i+1})$ depends on the parameters of the multi-component isotherm. Pe and St are the Peclet and Stanton numbers, respectively.

Using the hodograph transform, Rhee and Amundson [23] have also shown that a plot of C_i versus C_{i+1} is a straight line (solid line in Fig. 3), provided that these two components have the same apparent dispersion coefficient, D_a , and the same rate coefficient, k_f , in addition to the competitive Langmuir isotherm behavior. The equation of this straight line is

$$C_i = -\frac{C_{p,i}}{C_{p,i+1}} \cdot C_{i+1} + C_{p,i} \tag{13}$$

Therefore, the function $\mathcal{F}(C_i, C_{p,i}, C_{i+1}, C_{p,i+1})$ can be decoupled into two functions, $\mathcal{F}(C_i, C_{p,i}, C_{p,i+1})$ and $\mathcal{F}(C_{i+1}, C_{p,i}, C_{p,i+1})$, and the following equations give the SLT in displacement chromatography:

$$|\Delta\xi| = \frac{1 + K_d}{K_d} \left[\frac{D_a}{uL} + \frac{K_d u}{(1 + K_d)^2 k_f L} \right] \times \left| \frac{1 + \alpha}{1 - \alpha} \right| \ln \left| \frac{1 - \theta}{\theta} \right| \tag{14a}$$

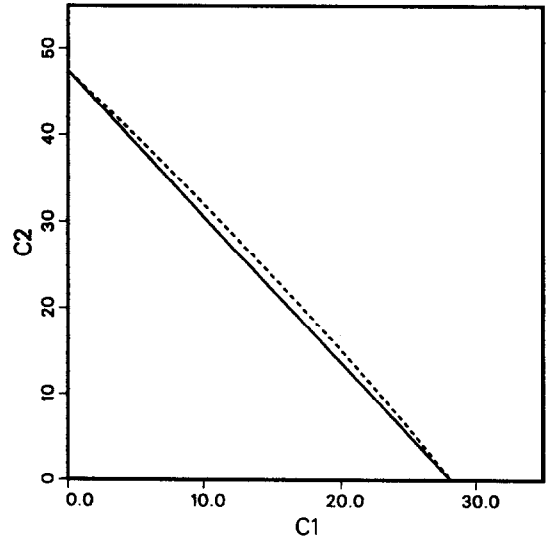


Fig. 3. Hodograph plot of the three chromatograms in Fig. 4. Solid line: $k_{t,1} = k_{t,2} = k_{t,d} = 20 \text{ min}^{-1}$ and $k_{t,1} = k_{t,2} = k_{t,d} = 200 \text{ min}^{-1}$. Dashed line: $k_{t,1} = 20 \text{ min}^{-1}$, $k_{t,2} = 200 \text{ min}^{-1}$, $k_{t,d} = 100 \text{ min}^{-1}$. Concentrations in mg/ml.

$$|\Delta\eta_x| = \frac{1 + K_d}{K_d} \left[\frac{D_a}{u} + \frac{K_d u}{(1 + K_d)^2 k_f} \right] \times \left| \frac{1 + \alpha}{1 - \alpha} \right| \ln \left| \frac{1 - \theta}{\theta} \right| \tag{14b}$$

$$|\Delta\eta_i| = \left[\frac{(1 + K_d)^2 D_a}{K_d u^2} + \frac{1}{k_f} \right] \left| \frac{1 + \alpha}{1 - \alpha} \right| \ln \left| \frac{1 - \theta}{\theta} \right| \tag{14c}$$

Hence, the SLT in displacement chromatography depends on the axial dispersion coefficient, the mass transfer coefficient and the separation factor, α , of the two adjacent band components, and on the injection concentration and retention factor, k'_0 of the displacer. The SLT does not depend on the retention factor or the feed concentration of either adjacent components. Obviously, when the isotachic train is formed, the concentration of each band reaches a plateau concentration which is determined by the concentration of the displacer.

The values of the SLT given by eqn. 14b are compared in Table I with those derived from chromatograms obtained by numerical integration of the system of equations of the transport-

TABLE I
SHOCK LAYER THICKNESS AND MASS TRANSFER COEFFICIENT

Parameter ^a	Shock layer thickness (min) ^b				Mass transfer coefficient k_t (min ⁻¹) ^c		
	1r	2f	2r	d	Component 1	Component 2	Displacer
N	2.175	2.175	2.020	2.020	20	20	20
T	2.033	2.033	2.033	2.033			
N	1.691	1.691	1.619	1.619	200	200	200
T	1.513	1.513	1.513	1.513			
N	1.918	1.905	1.640	1.640	20	200	100

^a N = SLT from numerical calculation; T = SLT from eqn. 14b.

^b 1r = Rear shock for the first component; 2f = front shock for the second component; 2r = rear shock for the second component; d = front shock for the displacer (see Fig. 2).

^c D_L 0.00137 cm²/min.

dispersive model [31]. The calculated chromatograms are shown in Fig. 4. It is seen in Table I that, when the two adjacent components have the same axial dispersion and kinetic coefficients, the SLT of the rear shock of the less retained band is the same as the SLT of the front shock of the more retained band. The SLT changes from pair to pair of components, but it is the same for all pairs having the same separation factor.

If the two adjacent components do not have

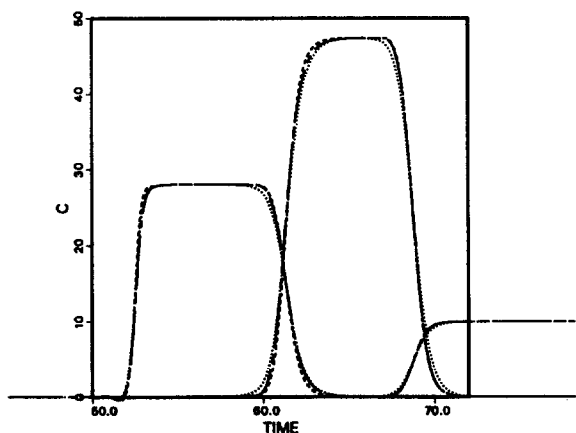


Fig. 4. Overlay of three displacement chromatograms ($\theta = 0.1$). Dotted line: $k_{t,1} = k_{t,2} = k_{t,d} = 20$ min⁻¹. Dashed line: $k_{t,1} = k_{t,2} = k_{t,d} = 200$ min⁻¹. Chain dashed line: $k_{t,1} = 20$, $k_{t,2} = 200$, $k_{t,d} = 100$ min⁻¹. The axial dispersion coefficient is the same for all the components, 0.00137 cm²/min. Langmuir isotherm parameters: $a_1 = 7.0$, $a_2 = 11.2$, $a_d = 17.9$, $b_1 = 0.07$, $b_2 = 0.079$, $b_d = 0.66$. Concentration in mg/ml, time in min.

the same axial dispersion and kinetic coefficients, the plot of C_i versus C_{i+1} is not a straight line (see chain line in Fig. 3), and the decoupling of $\mathcal{F}(C_i, C_{p,i}, C_{i+1}, C_{p,i+1})$ is not possible, even in the case of the competitive Langmuir isotherm model. In this case, no simple analytical solution for the SLT can be derived. However, the SLT will be between the two values calculated using eqn. 14 with the smaller k_t (e.g., 20 in Table I) and the larger k_t (200 in Table I). Therefore, a good approximation of the SLT can still be obtained from eqn. 14.

DISCUSSION

We now discuss the consequences of eqn. 14, the dependence of the SLT on the characteristics of the displacer, the mobile phase velocity and the separation factor.

Dependence of the SLT on the retention factor and the concentration of the displacer

The SLT in displacement chromatography does not depend directly on the retention factors of either the components involved or their concentrations, but it does depend on them indirectly, through K_d . The condition for achievement of the isotachic train is that the value of K (eqn. 3) is the same for all the components of the mixture, and for the displacer:

$$K_1 = K_2 = K_i = \dots = \frac{k'_{0,d}}{1 + \Gamma_d} \quad (15)$$

Hence, the plateau concentration of each component in the isotachic train is obtained by solving eqn. 15 for $C_{p,t}$. This explains why in displacement, unlike in frontal analysis, the retention factors and the concentrations of the sample components do not appear in eqn. 14.

Figs. 5 and 6 illustrate the dependence of the SLT on the retention factor, $k'_{0,d}$ (Fig. 5), and on the injection concentration, C_d (Fig. 6), of the displacer. We see (Fig. 5) that, as the retention factor of the displacer increases, the SLT decreases rapidly at first. Then, $\Delta\eta_x$ tends slowly towards 0 as $k'_{0,d}$ increases indefinitely. $\Delta\eta_t$, on the other hand, reaches a minimum beyond which it starts to increase slowly, with a slanted asymptote. Differentiation of eqn. 14c by respect to K_d shows that the optimum value of $k'_{0,d}$ for minimum SLT_t (SLT in time units) corresponds to

$$K_d = 1 \quad (16a)$$

$$k'_{0,d} = 1 + b_d C_d \quad (16b)$$

$$C_d = \frac{k'_{0,d} - 1}{b_d} \quad (16c)$$

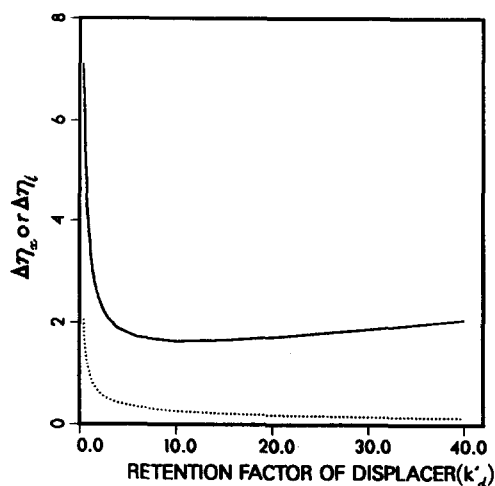


Fig. 5. Plot of the SLT versus the retention factor of the displacer, $k'_{0,d}$, based on eqn. 14b or 14c. $D_a = 0.00137 \text{ cm}^2/\text{min}$, $k_t = 20 \text{ min}^{-1}$, $u = 0.3 \text{ cm/min}$, $\theta = 0.05$, $C_d = 20 \text{ mg/ml}$, $\alpha = 1.5$. Langmuir isotherm parameter $b_d = 0.5 \text{ ml/mg}$. Dotted line: $\Delta\eta_x$ versus $k'_{0,d}$. Solid line: $\Delta\eta_t$ versus $k'_{0,d}$.

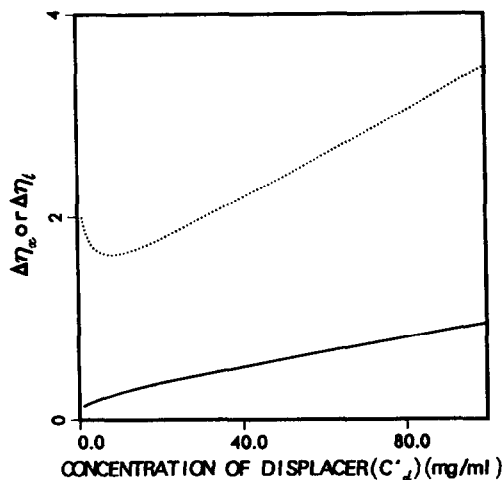


Fig. 6. Plot of the SLT versus the displacer concentration, C_d , based on eqn. 14b or 14c. $D_a = 0.00137 \text{ cm}^2/\text{min}$, $k_t = 20 \text{ min}^{-1}$, $u = 0.3 \text{ cm/min}$, $\theta = 0.05$, $k'_{0,d} = 5$, $\alpha = 1.5$. Langmuir parameter $b = 0.5 \text{ ml/mg}$. Solid line: $\Delta\eta_x$ versus C_d . Dotted line: $\Delta\eta_t$ versus C_d .

Depending whether the choice of the displacer is restricted or not, these equations define an optimum of the displacer retention factor (Fig. 5) or of its concentration (Fig. 6) for achieving minimum $\Delta\eta_t$, but there are no minima for $\Delta\eta_x$. As illustrated in Fig. 6, $\Delta\eta_x$ always increases with increasing displacer concentration. This result is important because it is just the opposite of what happens in single-component frontal analysis, where the SLT decreases with increasing feed concentration.

Dependence of the SLT on the mobile phase linear velocity

Fig. 7 shows three overlaid displacement chromatograms. The profiles of the isotachic train were calculated at three different mobile phase linear velocities. The values of $\Delta\eta_x$ and $\Delta\eta_t$ derived from these chromatograms are compared with those given by eqns. 14b and 14c in Figs. 8 and 9 respectively, showing the expected agreement. Fig. 8 shows the plot of $\Delta\eta_x$ versus the linear velocity, u . The curve obtained is very similar to the plot of the column HETP versus u in linear chromatography. Both curves exhibit a minimum, demonstrating the existence of an optimum mobile phase velocity for which the separation performance will be best.

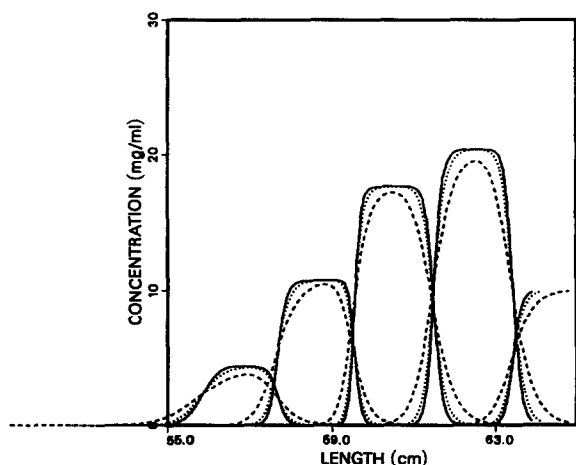


Fig. 7. Four-component displacement separations at three different mobile phase velocities. Chain line, $u = 2$ cm/min; dotted line, $u = 1$ cm/min; solid line, $u = 0.2$ cm/min. $D_a = 0.00137$ cm²/min, $k_f = 20$ min⁻¹. Langmuir isotherm parameters: $a_1 = 1.5$, $a_2 = 2.25$, $a_3 = 4.0$, $a_4 = 6.0$, $a_d = 9.0$, $b_1 = 0.04$, $b_2 = 0.07$, $b_3 = 0.12$, $b_4 = 0.18$, $b_d = 0.6$.

The dependence of D_a on the mobile phase velocity has been discussed abundantly in linear chromatography. We assume in this work that the molecular diffusion coefficients and the kinetic coefficients are independent of the concentration. The classical equations of Van Deem-

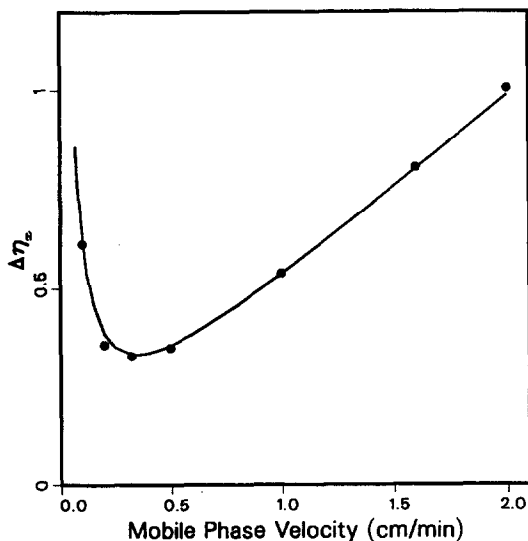


Fig. 8. $\Delta\eta_x$ versus mobile phase velocity. Solid line: calculation from eqn. 14b ($\alpha = 1.5$, $\theta = 0.05$). Symbols: $\Delta\eta_x$ from the numerical calculations in Fig. 7.

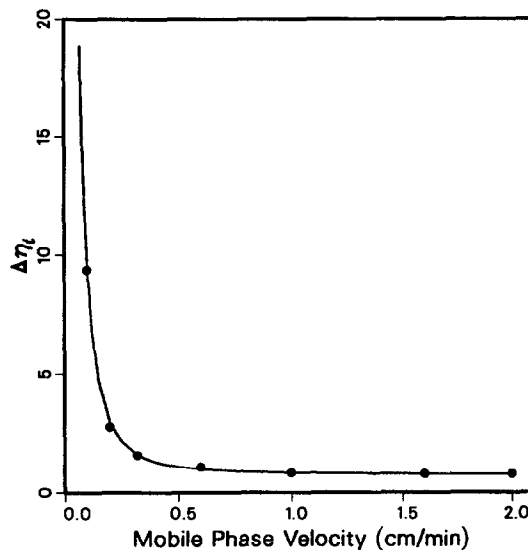


Fig. 9. Same as Fig. 8, but $\Delta\eta_t$ versus mobile phase velocity.

ter *et al.* [28] and Knox and Saleem [29] give, respectively,

$$2D_a = Au + B \quad (17a)$$

$$2D_a = B + Au^{1/3} \quad (17b)$$

We use here the former, Van Deemter equation. Differentiation of eqn. 14b with respect to u shows that the SLT is minimum for the following value of the linear velocity:

$$u_{\text{opt}}^S = \sqrt{\frac{D_a(1 + K_d)^2 k_f}{K_d}} \quad (18)$$

where K_d refers to the displacer and the coefficients D_a and k_f are those of the two components considered. Note that in the derivation of eqn. 14 it was assumed that these coefficients are the same for two successive components. Remarkably, this equation is otherwise the same as that found in frontal analysis. The optimum velocity for minimum shock layer thickness in displacement chromatography is the velocity for which the SLT of the displacer breakthrough curve is also a minimum. Both equations have for the limit when C_d tends towards zero the optimum linear velocity in linear chromatography.

We observe in Fig. 7 that the SLT of the front

shock of the first band is larger than the others. The reason is due to the difference in the dependence of single-component and binary SLT on the height of the concentration step. The opposite result could also be observed. Fig. 7 shows also that the SLT does not vary much as long as the linear velocity remains close to u_{opt}^S . However, unlike the band width in linear chromatography, the SLT broadens rapidly when the linear velocity increases well above u_{opt}^S . This explains the phenomenon reported by Horváth and co-workers [12,16] of the existence of an optimum in the purity of the product collected when the flow-rate is decreased. The flow-rate below which the purity degrades rapidly corresponds to the optimum velocity given by eqn. 18.

The optimum velocity depends also on the values of the axial dispersion coefficient, D_a , and the kinetic coefficient, k_f . However, apart from using small particles with well accessible pores, such as the packings developed for high-performance liquid chromatography, there is little we can do to achieve higher values of u_{opt}^S .

Dependence of the optimum velocity u_{opt}^S on the retention factor and concentration of the displacer

As shown by eqn. 18, the optimum velocity for minimum SLT depends on the K_d , and hence on the concentration and the retention factor of the displacer selected. The dependence of u_{opt}^S on the retention factor of the displacer is illustrated in Fig. 10. Differentiation of eqn. 18 shows that u_{opt}^S is minimum for

$$k'_{0,d} = 1 + \Gamma_d \tag{19}$$

u_{opt}^S is equal to u_{opt}^L , the optimum velocity for minimum HETP in linear chromatography, for

$$k'_{0,d} = \sqrt{1 + \Gamma_d} \tag{20}$$

We see in Fig. 10, where plots of u_{opt}^S and u_{opt}^L versus $k'_{0,d}$ are shown, that u_{opt}^S is larger than u_{opt}^L when $k'_{0,d}$ is smaller than $\sqrt{1 + b_d C_d}$. In practice, this value of the retention factor of the displacer is nearly impossible to achieve, since it is almost always lower than 2. $k'_{0,d} = 2$ requires $b_d C_d = 3$, a value of the displacer concentration for which the equilibrium concentration of the

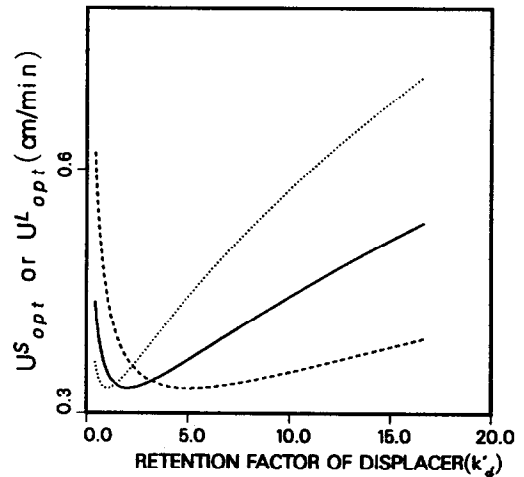


Fig. 10. Dependence of u_{opt}^S on $k'_{0,d}$ for two displacer concentrations, 20 mg/ml (solid line) and 40 mg/ml (dashed line). $b_d = 0.1$ ml/mg, $D_a = 0.00137$ cm²/min, $k_f = 20$ min⁻¹. The dotted line shows u_{opt}^L versus $k'_{0,d}$.

displacer in the stationary phase is $q = 3q_s/4$, or 75% of the saturation capacity. Hence, in almost all cases, u_{opt}^S is smaller than u_{opt}^L , and often much smaller. In almost all the reports on separations made with displacement chromatography, the displacers chosen have very large retention factors (about 10–15) [15,32]. This is why the optimum linear velocity u_{opt}^S in displacement is very low compared with the optimum linear velocity u_{opt}^L in linear chromatography, and why the production rate in displacement chromatography cannot be as large as anticipated.

Fig. 11 shows the dependence of u_{opt}^S on the concentration of the displacer for two values of its retention factor, 2 and 10. The dependence of u_{opt}^S on the displacer concentration is very different for the two values of $k'_{0,d}$ in the whole accessible concentration range, although both curves have minima. Rearrangement of eqn. 19 shows that $\Delta\eta_x$ is minimum for

$$K = 1 \tag{21a}$$

thus

$$C_d = \frac{k'_{0,d} - 1}{b_d} \tag{21b}$$

u_{opt}^S is equal to u_{opt}^L (cf., straight lines in Fig. 11) at the displacer concentration [24]

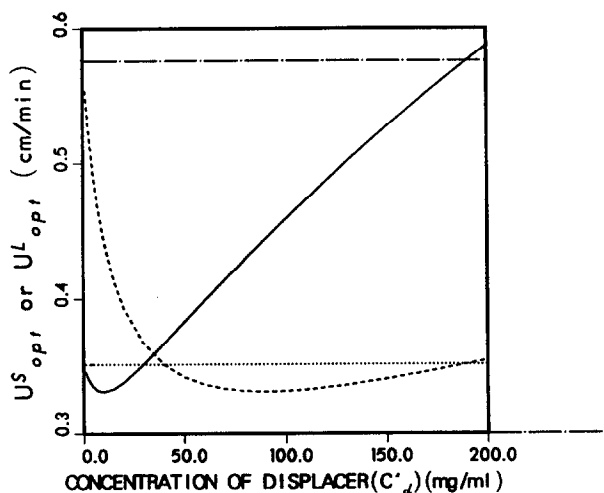


Fig. 11. Dependence of u_{opt}^S on the injection concentration of the displacer for two values of $k'_{0,d}$, 2 (solid line) and 10 (dashed line). $b_d = 0.1$ ml/mg, $D_s = 0.00137$ cm²/min, $k_t = 20$ min⁻¹. Dotted line: u_{opt}^L for $k'_0 = 2$. Chain dashed line: u_{opt}^L for $k'_0 = 10$.

$$C_d = \frac{k'_{0,d}{}^2 - 1}{b_d} \quad (22)$$

For the lower value of $k'_{0,d}$, the optimum velocity increases nearly linearly with increasing concentration above ca. 30 mg/ml. For the higher

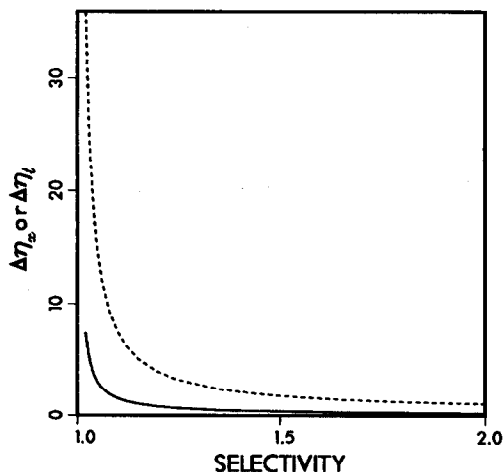


Fig. 12. Dependence of the SLT on the separation factor. $D_s = 0.00137$ cm²/min, $k_t = 20$ min⁻¹, $\theta = 0.05$, $K_d = 0.45$. Solid line: $\Delta\eta_x$ versus α . Dotted line: $\Delta\eta_t$ versus α .

value, in contrast, the optimum velocity remains nearly constant in that concentration range.

Dependence of the SLT on the separation factor

Fig. 12 shows the dependence of the SLT on the separation factor $\alpha = k'_2/k'_1$ of the two components. The SLT decreases dramatically as α increases from 1 to 1.3. For higher values of α , it slowly tends towards zero with increasing value of α .

CONCLUSIONS

This work has shown that the shock layer theory permits a detailed investigation of the optimization of the experimental conditions in displacement chromatography for the separation of multi-component mixtures which follow competitive Langmuir isotherm behavior. The theory explains the various empirical conclusions previously reported, including the apparent contradictions between results obtained under different conditions.

The conclusions of this work are extremely simple to apply, as it is not necessary to measure the isotherm parameters for the sample components. Only the single Langmuir isotherm parameters for the displacer and the separation factor at infinite dilution are required in all the equations. The results apply only to the isotachic train. Obviously, the production rate will be maximum if the sample size injected is such that this train is just formed when it reaches the outlet of the column.

Obviously, the conclusions of this work, like those of most of the theoretical work published so far in chromatography, assume that the column is radially homogeneous, hence that the column packing is homogeneous and perfect distribution of the sample takes place at the column inlet. These are ideal assumptions which have proved to be surprisingly satisfactory in liquid chromatography. Nevertheless, when preparative columns are used which have a diameter comparable to their length, new effects may arise owing to the random fluctuations which occur in the packing density. A detailed analysis of the phenomena which take place in actual columns will be published soon [33].

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